MATH 64
Test 1-SAMPLE

Name KEY

Use Algebraic Notation AND Show All of Your Work

Determine whether each relation is a function. (Circle the correct response.)
Give the domain and range for each relation.

[4, 6, 6 pts]

1a. $\{(4,5),(6,7),(8,8)\}$

Function OR Not a Function

Domain Set: $\{4,6,8\}$

Range Set: $\{5,7,8\}$

[4, 6, 6 pts]

1b. $\{(3,4),(3,5),(4,4),(4,5)\}$

Function OR Not a Function

Domain Set: $\{3,4\}$

Range Set: $\{4,5\}$

For $g(x)=2x^2+3x-1$*, find the indicated function values.*

[5 pts]

2. $g(0)=2(0)^2+3(0)-1$
$g(0)=0+0-1$
$g(0)=-1$

$g(0)=$ -1

[8 pts]

3. $g(-4)=2(-4)^2+3\cdot(-4)-1$
$g(-4)=2\cdot 16+(-12)-1$
$g(-4)=32-12-1$
$g(-4)=20-1$
$g(-4)=19$

$g(-4)=$ 19

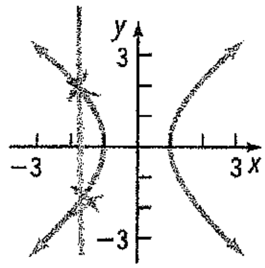
[10 pts]

4. $g(5a)=2(5a)^2+3(5a)-1$
$g(5a)=2\cdot 25a^2+15a-1$
$g(5a)=50a^2+15a-1$

$g(5a)=$ $50a^2+15a-1$

Identify graphs in which y is a function of x.
[6 pts each]

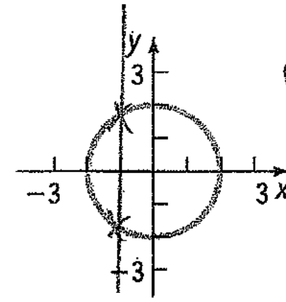
5.



Fails Vertical Line Test

Function OR Not a Function

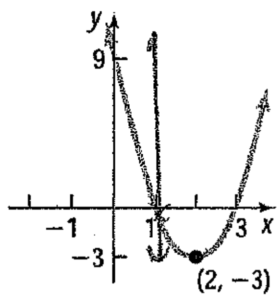
7.



Fails Vertical Line Test

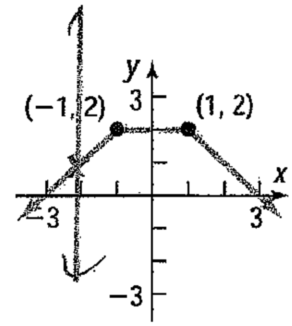
Function OR Not a Function

6.



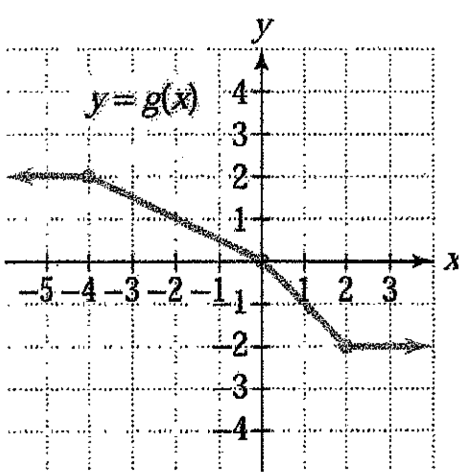
Function OR Not a Function

8.



Function OR Not a Function

Use the graph of g to find the following values. **[6 pts each]**



9. $g(-4)$

$g(-4)=$ 2

10. $g(2)$

$g(2)=$ −2

11. For what value of x is $g(x)=1$?

$x=$ −2

12. For what value of x is $g(x)=-1$?

$x=$ 1

[8, 8 pts]
13. (a) Explain how to determine whether a relation is a function.

Check that each domain element corresponds to exactly one range element. That is, no two ordered pairs in a function can have the same first component and different second components.

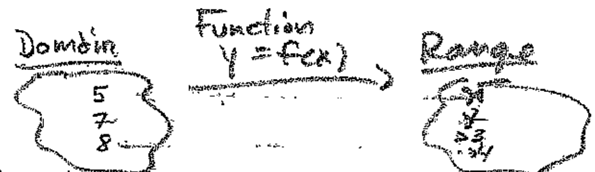
OR

Vertical Line Test

If any vertical line intersects a graph (of a relation) in more than one point, the graph does not define y as a function of x.

(b) What is a function?

A function is a correspondence from a first set, called the domain, to a second set, called the range, such that each element in the domain corresponds to exactly one element in the range.



[9 pts]
14. Which **one** of the following is true? (*Circle the correct letter.*)

(a) All relations are functions.

(b) No two ordered pairs of a function can have the same second component and different first components.

(c) The graph of every line is a function.

(d) A horizontal line can intersect the graph of a function in more than one point.

Find the domain of each function.

[8 pts]
15. $f(x) = 3x + 5$

All Real Numbers

Domain Set: $(-\infty, \infty)$

[10 pts]
16. $f(x) = \dfrac{2x}{x-3}$

$x - 3 = 0$

Excluded Value $\rightarrow$ $x = 3$

Domain Set: $\{x \mid x \neq 3\}$

For $g(x)=2x+7$ *and* $f(x)=3x^2-4x$, *find the indicated functions.*

[9 pts]

17. $(f+g)(x) = f(x) + g(x)$

$= (3x^2-4x) + (2x+7)$

$= 3x^2 - 4x + 2x + 7$

$= 3x^2 - 2x + 7$

$(f+g)(x) =$ $3x^2-2x+7$

[9 pts]

18. $(f-g)(x) = f(x) - g(x)$

$= (3x^2-4x) - (2x+7)$

$= 3x^2 - 4x - 2x - 7$

$= 3x^2 - 6x - 7$

$(f-g)(x) =$ $3x^2-6x-7$

[11 pts]

19. $(fg)(x) = [f(x)] \cdot [g(x)]$

$= (3x^2-4x) \cdot (2x+7)$

$= (3x^2)(2x) + (3x^2)(7) + (-4x)(2x) + (-4x)(7)$

$= 6x^3 + 21x^2 - 8x^2 - 28x$

$= 6x^3 + 13x^2 - 28x$

$(fg)(x) =$ $6x^3+13x^2-28x$

[9 pts]

20. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{3x^2-4x}{2x+7}$

$\left(\frac{f}{g}\right)(x) =$ $\frac{3x^2-4x}{2x+7}$

Solve each inequality, and state the solution set in INTERVAL notation.
Graph this solution set on a number line.

[8, 3, 4 pts]

21. $17-3x \le 29$

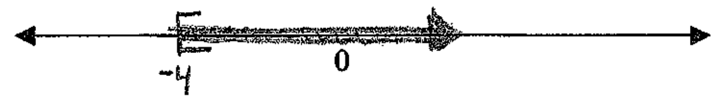
$-17+17-3x \le -17+29$

$-3x \le 12$

$-\frac{1}{3}\cdot\frac{(-3x)}{1} \ge -\frac{1}{3}\cdot\frac{12}{1}$

$x \ge -4$

$\{x \mid x \ge -4\}$



Solution Set: $[-4, \infty)$

[11, 3, 4 pts]

22. $3+2(3-2x) < 5(2-3x)$

$3+6-4x < 10-15x$

$9-4x < 10-15x$

$15x+9-4x < 15x+10-15x$

$11x+9 < 10$

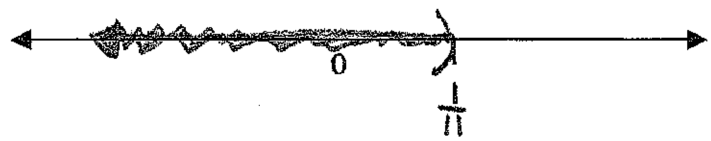
$-9+11x+9 < -9+10$

$11x < 1$

$\frac{1}{11}\cdot\frac{11x}{1} < \frac{1}{11}\cdot 1$

$x < \frac{1}{11}$

$\{x \mid x < \frac{1}{11}\}$



Solution Set: $(-\infty, \frac{1}{11})$

[16, 3, 4 pts]

23. $\dfrac{x-4}{6} \ge \dfrac{x-2}{9} + \dfrac{5}{18}$ LCD = 18

$\frac{18}{1}\left[\frac{x-4}{6}\right] \ge \frac{18}{1}\cdot\left[\frac{x-2}{9}+\frac{5}{18}\right]$

$3(x-4) \ge \frac{18}{1}\cdot\left(\frac{x-2}{9}\right)+\frac{18}{1}\cdot\frac{5}{18}$

$3x-12 \ge 2(x-2)+5$

$3x-12 \ge 2x-4+5$

$3x-12 \ge 2x+1$

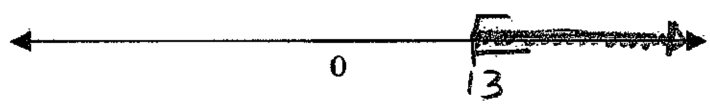
$-2x+3x-12 \ge -2x+2x+1$

$x-12 \ge 1$

$12+x-12 \ge 12+1$

$x \ge 13$

$\{x \mid x \ge 13\}$



Solution Set: $[13, \infty)$

[8, 4 pts]

24. When solving an inequality, under what conditions will it be necessary to **change the direction of the inequality symbol**? Give <u>one</u> example.

If we multiply or divide both sides of an inequality by the same negative quantity and reverse the direction of the inequality symbol, the resulting inequality is equivalent to the original one.

<u>Example:</u> $-5x \ge 15$

$-\frac{1}{5}\cdot\frac{-5x}{1} \le -\frac{1}{5}\cdot\frac{15}{1}$

$x \le -3$

Multiplying by a Negative

changes direction

Solve each compound inequality, and state the solution set in INTERVAL notation. Graph this solution set on a number line.

[16, 5, 6 pts]

25. $4(1-x)<-6$ AND $\frac{x-7}{5}\le -2$

$4-4x<-6$

$-4+4-4x<-6+(-4)$

$-4x<-10$

$\frac{-4x}{-4}>\frac{-10}{-4}$

$x>\frac{5}{2}$

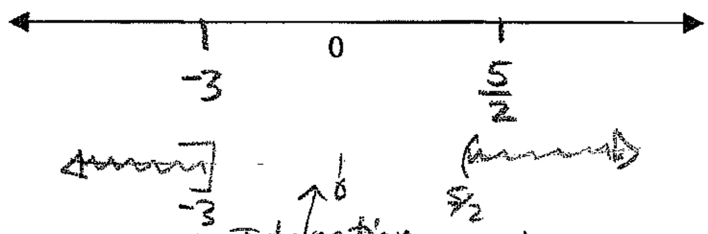
$\frac{5}{1}\cdot\left[\frac{x-7}{5}\right]\le 5(-2)$

$x-7\le -10$

$7+x-7\le -10+7$

$x\le -3$

"Set Intersection"



Solution Set: $\varnothing$

[16, 5, 6 pts]

26. $x-1\le 7x-1$ AND $4x-7<3-x$

$-x+x-1\le -x+7x-1$

$-1\le 6x-1$

$-1+1\le 6x-1+1$

$0\le 6x$

$\frac{0}{6}\le\frac{6x}{6}$

$0\le x$

$x+4x-7<3-x+x$

$5x-7<3$

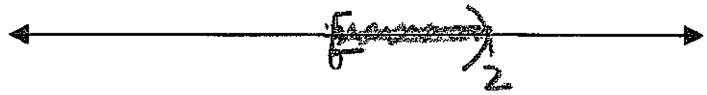
$7+5x-7<3+7$

$5x<10$

$\frac{5x}{5}<\frac{10}{5}$

$x<2$

"Set intersection"



$\{x \mid 0\le x<2\}$

Solution Set: $[0,2)$

[16, 5, 6 pts]

27. $4x+3<-1$ OR $2x-3\ge -11$

$-3+4x+3<-3+(-1)$

$4x<-4$

$\frac{4x}{4}<\frac{-4}{4}$

$x<-1$

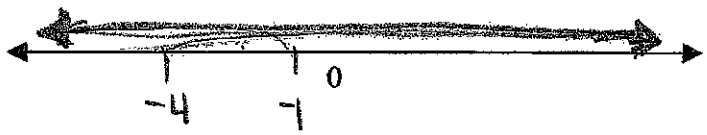
$3+2x-3\ge -11+3$

$2x\ge -8$

$\frac{2x}{2}\ge\frac{-8}{2}$

$x\ge -4$

"Set Union"



$\{x \mid$ all real numbers$\}$

Solution Set: $(-\infty,\infty)$

[16, 5, 6 pts]

28. $2x-5\le -11$ OR $5x+1\ge 6$

$5+2x-5\le -11+5$

$2x\le -6$

$\frac{2x}{2}\le\frac{-6}{2}$

$x\le -3$

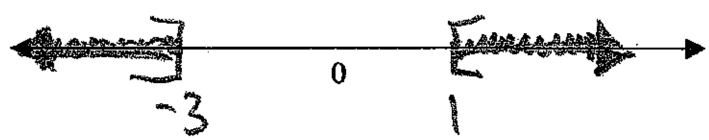
$-1+5x+1\ge -1+6$

$5x\ge 5$

$\frac{5x}{5}\ge\frac{5}{5}$

$x\ge 1$

"Set Union"



Solution Set: $(-\infty,-3]\cup[1,\infty)$

Solve each equation, and state the solution set.

[18, 5 pts]

29. $|2x-1|=7$

$-(2x-1)=7$, or $+(2x-1)=7$

$-2x+1=7$
$-2x+1-1=7-1$
$-2x=6$
$\frac{-2x}{-2}=\frac{6}{-2}$
$x=-3$

$2x-1+1=7+1$
$2x=8$
$\frac{2x}{2}=\frac{8}{2}$
$x=4$

check
$|2(-3)-1|=7$
$|-6-1|=7$
$|-7|=7$
$7=7$
TRUE!

check
$|2(4)-1|=7$
$|8-1|=7$
$|7|=7$
$7=7$
TRUE!

Solution Set: $\{-3,4\}$

[18, 5 pts]

30. $|x+1|+5=3$

$-5+|x+1|+5=-5+3$
$|x+1|=-2$

Distance From zero can't be negative!

$-(x+1)=-2$, or $+(x+1)=-2$

$-x-1=-2$
$-x-1+1=-2+1$
$-x=-1$
$-1(-x)=-1(-1)$
$x=1$

$x+1-1=-2-1$
$x=-3$

check
$|(1)+1|+5=3$
$|2|+5=3$
$2+5=3$
$7=3$
False!

check
$|(-3)+1|+5=3$
$|-2|+5=3$
$2+5=3$
$7=3$
False!

No Solution

Solution Set: $\emptyset$ or $\{\ \}$

[20, 5 pts]

31. $|6y-2|+4=32$

$-4+|6y-2|+4=-4+32$
$|6y-2|=28$

$-(6y-2)=28$, or $+(6y-2)=28$

$-6y+2=28$
$-6y+2-2=28-2$
$-6y=26$
$\frac{-6y}{-6}=\frac{26}{-6}$
$y=-\frac{13}{3}$

$6y-2+2=28+2$
$6y=30$
$\frac{6y}{6}=\frac{30}{6}$
$y=5$

check
$|6(-\frac{13}{3})-2|+4=32$
$|2(-13)-2|+4=32$
$|-26-2|+4=32$
$|-28|+4=32$
$28+4=32$
$32=32$
TRUE!

check
$|6(5)-2|+4=32$
$|30-2|+4=32$
$|28|+4=32$
$28+4=32$
$32=32$
TRUE!

Solution Set: $\{-\frac{13}{3}, 5\}$

[20, 5 pts]

32. $|2x-4|=|x-1|$

$-(2x-4)=x-1$, or $+(2x-4)=x-1$

$-2x+4=x-1$
$-2x+4+2x=2x+x-1$
$4=3x-1$
$4+1=3x-1+1$
$5=3x$
$\frac{5}{3}=\frac{3x}{3}$
$\frac{5}{3}=x$

$2x-4=x-1$
$2x-4-x=x-1-x$
$x-4=-1$
$x-4+4=-1+4$
$x=3$

check
$|2(\frac{5}{3})-4|=|(\frac{5}{3})-1|$
$|\frac{10}{3}-4|=|\frac{5}{3}-\frac{3}{3}|$
$|\frac{10}{3}-\frac{12}{3}|=|\frac{2}{3}|$
$|-\frac{2}{3}|=\frac{2}{3}$
$\frac{2}{3}=\frac{2}{3}$
TRUE!

check
$|2(3)-4|=|(3)-1|$
$|6-4|=|2|$
$|2|=|2|$
$2=2$
TRUE!

Solution Set: $\{\frac{5}{3}, 3\}$

Solve each absolute value inequality, and state the solution set in INTERVAL notation. Graph this solution set on a number line.

[20, 5, 6 pts]

32. $\left|\frac{3a-3}{4}\right| \le 6$

$-6 \le \frac{3a-3}{4} \le 6$

$-6 \cdot 4 \le 4 \cdot \left(\frac{3a-3}{4}\right) \le 4 \cdot 6$

$-24 \le 3a-3 \le 24$

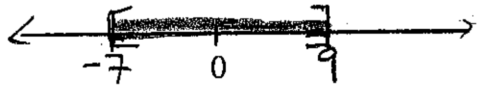
$-24+3 \le 3a-3+3 \le 3+24$

$-21 \le 3a \le 27$

$\frac{-21}{3} \le \frac{3a}{3} \le \frac{27}{3}$

$-7 \le a \le 9$

$\{a \mid -7 \le a \le 9\}$



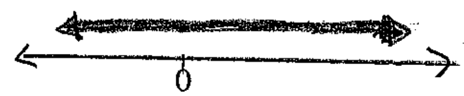
Solution Set: $[-7, 9]$

[18, 5, 6 pts]

33. $|w+4| > -12$

Absolute value is a distance. Distances are always greater than a negative number. This statement is always true. The solution is All Real numbers.

All Real Numbers



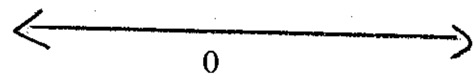
Solution Set: $(-\infty, \infty)$

[18, 5, 6 pts]

34. $|b-3| < -2$

Absolute value is a distance. Distances can never be less than a negative number. This statement is always false.

No Solution



The solution set is empty.

Solution Set: $\emptyset$

[23, 5, 6 pts]

35. $3|2y-1|+2 \ge 8$

$3|2y-1|+2-2 \ge 8-2$

$3|2y-1| \ge 6$

$\frac{3|2y-1|}{3} \ge \frac{6}{3}$

$|2y-1| \ge 2$

$-(2y-1) \ge 2$, or $+(2y-1) \ge 2$

$-2y+1 \ge 2$

$-2y+1-1 \ge 2-1$

$-2y \ge 1$

$\frac{-2y}{-2} \le \frac{1}{-2}$

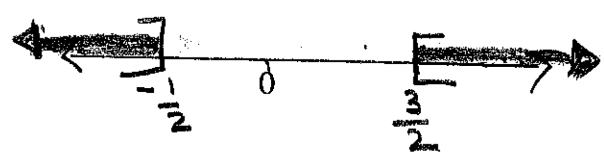
$y \le -\frac{1}{2}$

$2y-1+1 \ge 2+1$

$2y \ge 3$

$\frac{2y}{2} \ge \frac{3}{2}$

$y \ge \frac{3}{2}$



$\{y \mid y \le -\frac{1}{2} \text{ or } y \ge \frac{3}{2}\}$

Solution Set: $(-\infty, -\frac{1}{2}] \cup [\frac{3}{2}, \infty)$

For $f(x)=4x-3$, *and* $g(x)=5x^2-2$, *find the following:*
(Be sure to state your result in simplified form.)

[16 pts]

36. $(f\circ g)(x) = f(g(x))$
$= 4\cdot(g(x)) - 3$
$= 4(5x^2-2) - 3$
$= 20x^2 - 8 - 3$
$= 20x^2 - 11$

SDWK
$4\cdot(5x^2-2) = 4\cdot 5x^2 - 4\cdot 2$
$= 20x^2 - 8$

$(f\circ g)(x) =$ $20x^2-11$

[16 pts]

37. $(g\circ f)(x) = g(f(x))$
$= 5\cdot[f(x)]^2 - 2$
$= 5\cdot[4x-3]^2 - 2$
$= 5\cdot[(4x-3)(4x-3)] - 2$
$= 5[16x^2-24x+9] - 2$
$= 80x^2 - 120x + 45 - 2$
$= 80x^2 - 120x + 43$

SDWK
$(4x-3)^2 = (4x-3)(4x-3)$
$= (4x)(4x) + (4x)(-3) + (4x)(-3) + (-3)(-3)$
$= 16x^2 - 12x - 12x + 9$
$= 16x^2 - 24x + 9$

$5(16x^2-24x+9)$
$= 5\cdot 16x^2 + 5(-24x) + 5\cdot 9$
$= 80x^2 - 120x + 45$

$(g\circ f)(x) =$ $80x^2-120x+43$

[14 pts]

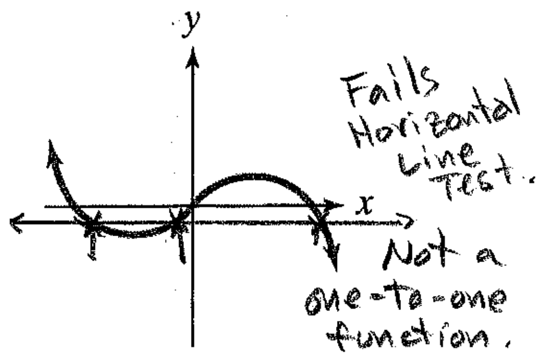
38. $(f\circ g)(2) = f(g(2))$
$= f(18)$
$= 4\cdot(18) - 3$
$= 72 - 3$
$= 69$

SDWK
$g(2) = 5(2)^2 - 2$
$g(2) = 5\cdot 4 - 2$
$g(2) = 20 - 2$
$g(2) = 18$

$(f\circ g)(2) =$ 69

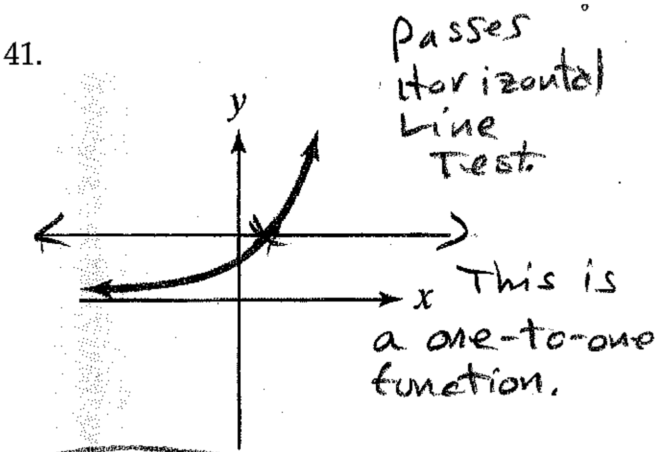
Identify the graphs that represent functions that are one-to-one and that have inverse functions. (Circle the correct response.)
[5 pts each]

39.



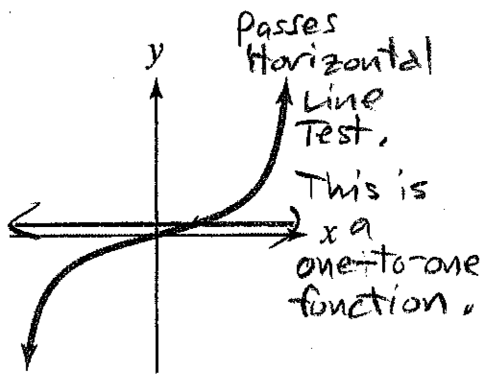
Has an Inverse Function, OR Does Not have an Inverse Function

41.



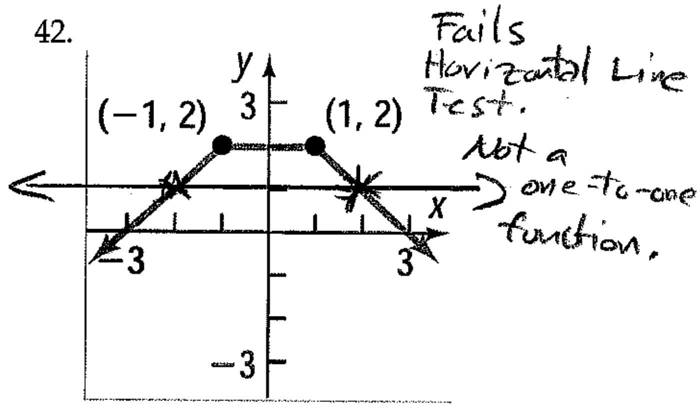
Has an Inverse Function, OR Does Not have an Inverse Function

40.



Has an Inverse Function, OR Does Not have an Inverse Function

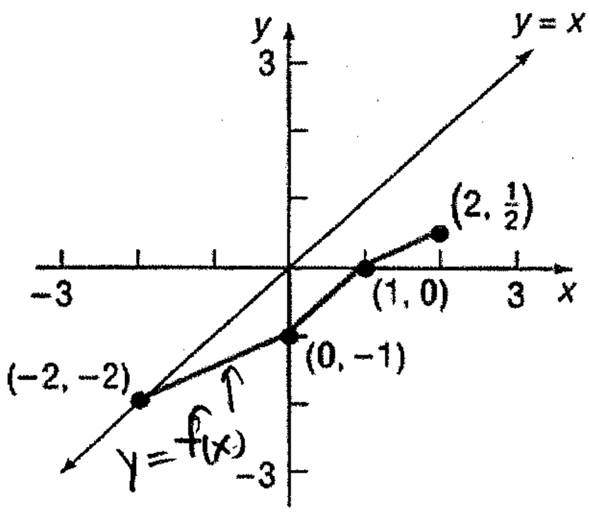
42.

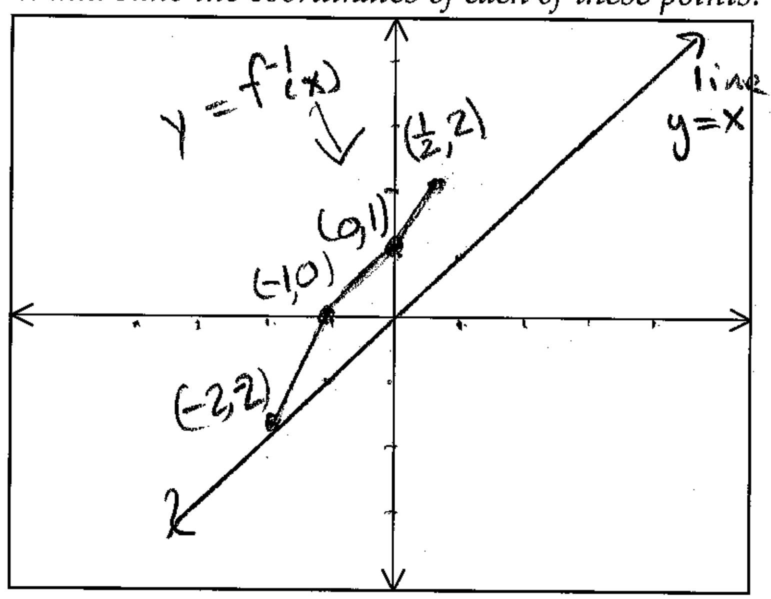


Has an Inverse Function, OR Does Not have an Inverse Function

The graph of a one-to-one function f is illustrated below. Draw the graph of the inverse function f^{-1}, and plot at least 4 points. Label and state the coordinates of each of these points.
[16, 4 pts]

43.





[20, 12, 12 pts]

44. *For the on-to-one function* $f(x) = 4x - 3$,

(a) *find an equation for* $f^{-1}(x)$, *the inverse function, then*

(b) *verify that your equation is correct by showing that* $f\left(f^{-1}(x)\right) = x$, *and* $f^{-1}\left(f(x)\right) = x$.

(Be careful with your notation and show your steps.)

(a) *find the inverse of* $f(x) = 4x - 3$

$$y = 4x - 3$$

Exchange x & y: $x = 4y - 3$

solve for y:

$$x + 3 = 4y - 3 + 3$$

$$x + 3 = 4y$$

$$\frac{x+3}{4} = \frac{4y}{4}$$

$$\frac{x+3}{4} = y \longrightarrow f^{-1}(x) = \frac{x+3}{4}$$

(a) $f^{-1}(x) =$ $\frac{x+3}{4}$

(b) *Show* $f\left(f^{-1}(x)\right) = x$

$$f(f^{-1}(x)) = 4\,[f^{-1}(x)] - 3$$

$$= 4\left[\frac{x+3}{4}\right] - 3$$

$$= (x+3) - 3$$

$$= x \quad \checkmark$$

(b) *Show* $f^{-1}\left(f(x)\right) = x$

$$f^{-1}(f(x)) = \frac{[f(x)] + 3}{4}$$

$$= \frac{[4x - 3] + 3}{4}$$

$$= \frac{4x}{4}$$

$$= x \quad \checkmark$$